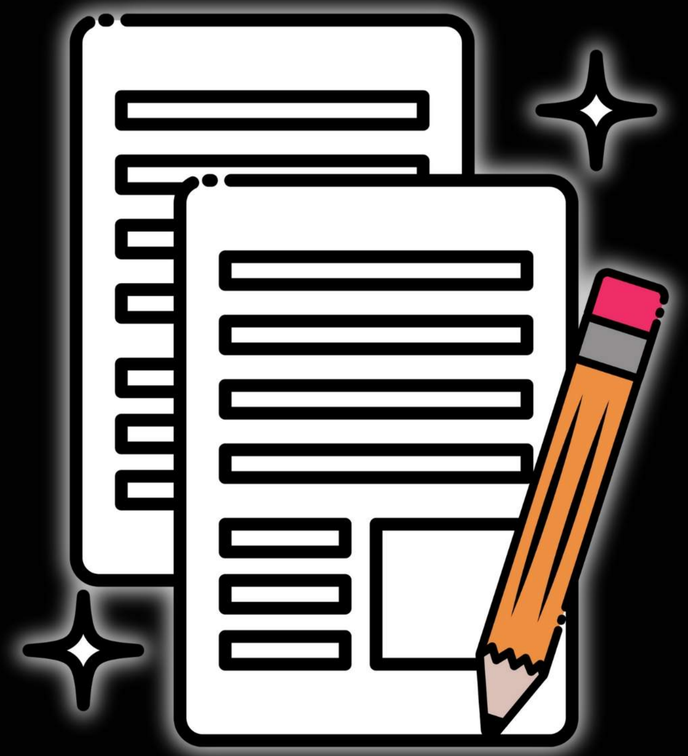
Most EXPECTED Questions



RELATIONS & FUNCTIONS

Most Expected Questions

A relation R is defined on a set of real numbers $\mathbb{R}$ as

$R = \{(x, y) : x \cdot y \text{ is an irrational number}\}$.

Check whether R is reflexive, symmetric and transitive or not.

Let $A = \{a, b, c\}$ and the relation R be defined on A, as follows: $R = \{(a, a), (b, c), (a, b)\}$. Then, write minimum number of ordered pairs to be added in R to make R reflexive and transitive. [NCERT Exemplar] **(2 marks)**

Show that the relation S in the set R of real numbers defined as $S = \{(a, b) : a, b \in R \text{ and } a \leq b^3\}$ is neither reflexive nor symmetric nor transitive. [Board Question]

Show that the function $f : R \to \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}, x \in R$ is one-one and onto function. [NCERT] **(5 marks)**

Let N denote the set of all natural numbers and R be the relation on N × N defined by (a, b) R $(c, d) \Leftrightarrow ad(b + c) = bc(a + d)$. Check whether R is an equivalence relation on N × N.

CHECK ONE-ONE ONTO

$f : R \to R$, $f(x) = x^2$

$f : N \to N$, $f(x) = x^2$

$f : R \to R$, $f(x) = x^3$

$f : N \to N$, $f(x) = x^3$

INVERSE TRIGONOMETRY

Most Expected Questions

Find the value of $\sin^{-1}(\sin\frac{3\pi}{5})$

$$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x},\ x\neq 0$$

Find the principal value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$.

What is the domain of the function $\cos^{-1}(2x-3)$?

(*a*) [–1, 1] **(*b*) (1, 2)**

(*c*) (–1, 1) **(*d*) [1, 2]**

$$\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\ -\frac{1}{\sqrt{2}}\le x\le 1$$

$$\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$$

MATRICES

Most Expected Questions

If for the matrix $A = \begin{bmatrix} \tan x & 1 \\ -1 & \tan x \end{bmatrix}$, $A + A' = 2\sqrt{3}\, I$, then the value of $x \in \left[0, \frac{\pi}{2}\right]$ is :

(A) 0 (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Let $A = [a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = \hat{j} - 2\hat{i}$. Then which of the following is true ?

(A) $a_{12} > 0$ (B) all $a_{ij} < 0$

(C) $a_{13} + a_{31} = -6$ (D) $a_{23} > a_{32}$

If $A = \begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix}$,

find the value of $(a + x) - (b + y)$.

If A and B are square matrices of same order, then $(AB^T - BA^T)$ is a

(A) symmetric matrix (B) skew-symmetric matrix

(C) null matrix (D) unit matrix

Number of symmetric matrices of order 3×3 with each entry 1 or -1 is

(A) 512 (B) 64

(C) 8 (D) 4

Let A be a matrix of order $m \times n$ and B is a matrix such that A^TB and BA^T are defined. Then, the order of B is :

(A) $m \times m$ (B) $n \times n$

(C) $m \times n$ (D) $n \times m$

DETERMINANTS

Most Expected Questions

If A is a square matrix of order 2 such that det (A) = 4, then det (4 adj A) is equal to :

(A) 16 (B) 64

(C) 256 (D) 512

Assertion (A) : For matrix $A = \begin{bmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{bmatrix}$, where $\theta \in [0, 2\pi]$, $|A| \in [2, 4]$.

Reason (R) : $\cos\theta \in [-1, 1], \forall\, \theta \in [0, 2\pi]$.

If $\left|\frac{A^{-1}}{2}\right| = \frac{1}{k|A|}$, where A is a 3×3 matrix, then the value of k is :

(a) $\frac{1}{8}$ (b) 8

(c) 2 (d) $\frac{1}{2}$

If $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find AB and use it to solve the following system of equations :

$x - 2y = 3$

$2x - y - z = 2$

$-2y + z = 3$

The equation of the path traversed by the ball headed by the footballer is $y = ax^2 + bx + c; (where\ 0 \le x \le 14\ and\ a, b, c \in R\ and\ a \neq 0)$ with respect to a XY-coordinate system in the vertical plane. The ball passes through the points $(2,15)$, $(4,25)$ and $(14,15)$. Determine the values of a, b and c by solving the system of linear equations in a, b and c, using matrix method. Also find the equation of the path traversed by the ball.

CONTINUITY & DIFFERENTIABILITY

Most Expected Questions

Differentiate $\tan^{-1}\dfrac{\sqrt{1-x^2}}{x}$ w.r.t. $\cos^{-1}(2x\sqrt{1-x^2})$, $x \in \left(\dfrac{1}{\sqrt{2}}, 1\right)$

If $x^p \cdot y^q = (x+y)^{p+q}$, prove that (*i*) $\dfrac{dy}{dx} = \dfrac{y}{x}$ and (*ii*) $\dfrac{d^2y}{dx^2} = 0$.

Find $\dfrac{dy}{dx}$, if $y = x^{\tan x} + \dfrac{\sqrt{x^2+1}}{2}$.

Show that the function $f(x) = |x-1| + |x+1|$, for all $x \in \mathbf{R}$, is not differentiable at the points $x = -1$ and $x = 1$. [CBSE 2015]

Determine the values of a, b and c for which the function

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x}, & \text{if } x < 0 \\ c, & \text{if } x = 0 \\ \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{b\sqrt{x^3}}, & \text{if } x > 0 \end{cases}$$

may be continuous at $x = 0$.

APPLICATIONS OF DERIVATIVE

Most Expected Questions

The relation between the height of the plant (y cm) with respect to exposure to sunlight is governed by the equation $y = 4x - \frac{1}{2}x^2$, where x is the number of days exposed to sunlight.

(i) Find the rate of growth of the plant with respect to sunlight.

(ii) In how many days will the plant attain its maximum height ? What is the maximum height ?

A jet of an enemy is flying along the curve $y = x^2 + 2$. A soldier is placed at the point (3, 2). What is the nearest distance between the soldier and the jet ?

Of all the closed right circular cylindrical cans of volume 128 π cm^3, find the dimensions of the can which has minimum surface area.

Find the intervals in which the function $f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12$ is (*a*) strictly increasing (*b*) strictly decreasing.

A wire of length 34 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a rectangle whose length is twice its breadth. What should be the lengths of the two pieces, so that the combined area of the square and the rectangle is minimum?

INTEGRALS

Most Expected Questions

Find :

$$\int \frac{\cos x}{(4+\sin^2 x)(5-4\cos^2 x)}\,dx$$

$$\int_0^{\frac{3}{2}} |\, x\cos \pi x \,|\, dx$$

$$\int \frac{1}{\cos(x-a)\cos(x-b)}\,dx$$

Evaluate $\int \frac{x+2}{\sqrt{x^2+5x+6}}\,dx$

Evaluate $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cdot \cos^2 x}\,dx$

APPLICATION OF INTEGRALS

Most Expected Questions

Find the area of the region $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$, using integration.

Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration.

Using integration, find the area of the region bounded by the line $y = 5x + 2$, the x – axis and the ordinates $x = -2$ and $x = 2$.

Using integration, find the area of the region bounded by the triangle whose vertices are (–1, 2), (1, 5) and (3, 4).

Sketch a graph of $y = x^2$. Using integration, find the area of the region bounded by $y = 9$, $x = 0$ and $y = x^2$.

DIFFERENTIAL EQUATIONS

Most Expected Questions

Write the sum of the order and degree of the following differential equation: $\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\} = 0.$

$$\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right]\frac{dx}{dy} = 1,\ x \neq 0$$

Solve the differential equation

$$x\frac{dy}{dx} = y(\log y - \log x + 1).$$

Solve the following differential equation:

$$y^2dx + (x^2 - xy + y^2)dy = 0$$

Find the general solution of the differential equation

$$y\,dx - (x + 2y^2)\,dy = 0.$$

VECTOR ALGEBRA

Most Expected Questions

Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Evaluate the quantity $\mu = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, If $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{c}| = 2$.

Find a vector $\vec{r}$ of magnitude $3\sqrt{2}$ units which make an angle of $\frac{\pi}{4}$ and $\frac{\pi}{2}$ with y and z-axes respectively.

If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then prove that $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$.

1. (a) If vectors $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{b} + \lambda\vec{c}$ is perpendicular to $\vec{a}$, then find the value of λ.

3D GEOMETRY

Most Expected Questions

If a line makes angles of 90°, 135° and 45° with the x, y and z axes respectively, then its direction cosines are

(a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

(b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$

(d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

Find the length and foot of perpendicular drawn from the point (2, – 1, 5) to line

$$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$$

Show that the lines $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$; $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$ are intersecting. Hence, find their point of intersection.

Find the value of λ, so that following lines are perpendicular to each other.

$$\frac{x+5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1} \text{ and } \frac{x}{1} = \frac{2y+1}{4\lambda} = \frac{1-z}{-3}$$

Find the shortest distance between the lines L_1 and L_2 given below

L_1 : The line passing through $(2, -1, 1)$ and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$

$L_2 : \vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}$.

LINEAR PROGRAMMING

Most Expected Questions

The objective function for a given linear programming problem is $Z = ax + by - 5$. If Z attains same value at (1, 2) and (3, 1), then

(a) $a + 2b = 0$ **(b)** $a + b = 0$

(c) $a = b$ **(d)** $2a - b = 0$

Maximise $Z = -x + 2y$, subject to the constraints:

$x \geq 3, x + y \geq 5, x + 2y \geq 6, y \geq 0.$

Minimise $Z = 3x + 5y$

such that $x + 3y \geq 3, x + y \geq 2, x, y \geq 0.$

Determine graphically the minimum value of the objective function

$$Z = -50x + 20y$$

subject to the constraints:

$$2x - y \geq -5$$
$$3x + y \geq 3$$
$$2x - 3y \leq 12$$
$$x \geq 0, y \geq 0$$

Consider the following Linear Programming Problem:

Minimise $Z = x + 2y$

Subject to $2x + y \geq 3, x + 2y \geq 6, x, y \geq 0.$

Show graphically that the minimum of Z occurs at more than two points.

PROBABILITY

Most Expected Questions

If $P(A) = \frac{2}{5}$, $P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$, then $P(A'|B') \cdot P(B'|A')$ is equal to :

[NCERT Exemplar]

(a) $\frac{5}{6}$

(b) $\frac{5}{7}$

(c) $\frac{25}{42}$

(d) 1

By examining the chest X-ray, the probability that TB is detected when a person is actually suffering from it is 0.99. The probability that the doctor diagnosis incorrectly that a person has TB on the basis of X-ray is 0.001. In a certain city, 1 in 1000 persons suffers from TB. A person is selected at random and is diagnosed to have TB. What is the chance that he actually has TB?

A letter is known to have come either from TATANAGAR or CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probability that the letter has come from CALCUTTA?

An urn contains m white and n black balls. A ball is drawn at random and is put back into the urn along with k additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. Show that the probability of drawing a white ball does not depend on k.

Three rotten apples are mixed with seven fresh apples. Find the probability distribution of the number of rotten apples, if three apples are drawn one by one with replacement. Find the mean of the number of rotten apples.